Notes on Some aspects of unsteady hydraulics of watercourses

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Notes on

Some aspects of unsteady hydraulics of watercourses

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Preface

These notes is prepared for the support of projects and courses about open channel flows in watercourses at Aalborg University. It is thought as continuation of *Kompendium i Vandløbshydraulik – stationære strømninger* (Compendium in Hydraulics of Watercourse – steady flows) which is available in Danish. The intention of the notes is to prepare the reader for the use of the advanced commercial computer models available for solving hydraulic problems in watercourses and open channels. It is definitely not the intention to present the complete hydraulic theory behind those computer models. The specific details can be found in the literature and in the respective *reference manuals* for the computer models.

The notes are under continuous development and any comments are welcome. Ask for latest version.

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List of content

1. The unsteady flow equations for watercourses

From a theoretical point of view steady state flows never exist in real watercourses. Nevertheless, the assumption of steady state conditions is a very useful simplification for the description of large part of hydraulic phenomena. Quite often a so-called *semi steady state* approach (or a *worst case scenario*) can be used especially for design of artificial channels and pipelines (storm sewers). However, several examples can be given where the steady state assumption leads to an over-simplification of the flow situation and where a non-steady approach is necessary to explain the observed flow situation. A system analysis based on force and/or energy balance can often be useful for the choice of simplification for the actual case.

General

Stationary, one-dimensional flow problems in watercourses may often be solved simply by paper and pencil (or spreadsheet) because the solution is restricted in amount only to depend of the longitudinal coordinate (x-coordinate). When the flow becomes non-stationary, the complexity increases with one further dimension and the number of calculations increases considerably. Accordingly it is no longer practical to solve the problems by hand and, consequently, non-steady problems are almost always solved using computer models.

In the field of stationary hydraulics, problems are most often solved with the energy equation together with the continuity equation. In the non-stationary hydraulics, where the flow accelerates or decelerates, we normally use the momentum equation and the continuity equation. The momentum equation can also be understood as the “flow-version” of Newton´s second law.

It should be mentioned that a non-stationary version of the energy equation exists, but this is less convenient to use. The energy equation is formally derived from the momentum equation by integration in time, and because the solution of a non-steady flow case itself is also, in principle, an integration in time it is obvious that an application of the energy equation here implies an unnecessary complication of the matter.

The Saint-Venant equations

These notes only deal with one-dimensional flows in watercourses where flow velocity and surface level only depend on one longitudinal coordinate x. These non-stationary flow equations (the momentum and continuity equations together) are normally designated the *Saint-Venant equations.* (Jean Claude Saint-Venant, 1797-1886).

In the following, these equations are discussed more closely. Several formulations exist. For convenience and simplicity, the equations given here correspond to a prismatic channel, where the channel cross-section is unchanged (or vary slowly) in the longitudinal direction. We focus on a vertical slice perpendicular to the flow direction, which has the flow area $A = A(x,t)$ and an infinitesimal length of dx (Figure 1-1)

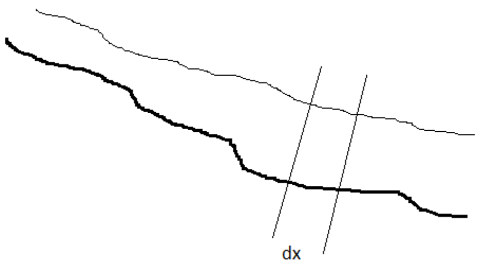

Figure 1-1 Basic slice element

The continuity equation

The non-stationary continuity equation expresses that a change in the longitudinal flow Q equals a change in the cross sectional area A. If also a lateral inflow q_L (flow per unit length) takes place, the general formulation is as follows

$$\frac{\partial Q}{\partial t} + \frac{\partial A}{\partial x} = q_L \quad (1)$$

For prismatic channels with the surface width B the continuity equation is as follows:

$$\frac{\partial Q}{\partial t} + B\,\frac{\partial y}{\partial x} = q_L \quad (2)$$

The momentum equation

As mentioned, the momentum equation can be understood as special formulation of the longitudinal component of Newton´s second law. Here, we meet a fundamental problem in hydrodynamics. The complication is that Newton´s second law is valid for a specific volume of water which moves with the velocity of the volume whereas we want a formulation for our modelling which can describe the flow in a fixed coordinate system in space which does not move and in which we want to calculate flow Q (or velocity V) and head h (or water depth y) as function of x and t. In this fixed coordinate system, a water slice will experience a *total acceleration DV/Dt* which can be written as

$$\frac{DV}{Dt} = \frac{\partial V}{\partial x} + V\,\frac{\partial V}{\partial x} \quad (3)$$

where V is the average velocity over the cross-section.

Equation (3) could be interpreted as the water slice will have both a *local acceleration* $\partial V/\partial t$ because the velocity changes locally and a *convective acceleration* $V\ \partial V/\partial x$ because the slice is moving along to a place where the velocity is different. For this reason, the momentum equation (4) contains to acceleration terms (terms a and b):

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\,\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\,\frac{\partial y}{\partial x} - g\,\left(I_0 - I_f\right) = 0 \qquad (4)$$

(a) + (b) + (c) + (d) + (e) = 0

where y is the water depth, $\left(I_0 - I_f\right) = \frac{\partial h}{\partial x}$ is the head slope, $I_0 = \frac{dz}{dx}$ is the bottom slope (constant in time) and I_f is the friction slope for example given by the Manning formula $Q = A\,M\,R^{2/3}\,I_f^{1/2}$ (R *is* hydraulic radius).

The individual terms in the momentum equation can be understood as longitudinal forces acting parallel to the bottom line. The terms are denoted as seen in the following Table 1.

Table 1 Elements in themomentum equation

Term	**Name**
(a)	Local acceleration
(b)	Advective acceleration
(c)	Longitudal pressure gradient
(d)	Bottom gradient
(e)	Friction gradient

In practical cases some of the terms in the momentum equationcan be excluded when solving the actual problem. Table 2 lists the names of the simplified approxiations.

Tabel 2 Simplified equations

Terms included	Name of approximation
(d) + (e)	Kinematic wave
(c) + (d) + (e)	Diffusive wave
(a) + (b) + (c) + (d) + e)	Full dynamic wave

In connection with watercourses, computer models building on “full dynamic wave approximation” are often used also for steady state calculations (e.g. the DHI MIKE Hydro model), which in principle should imply the terms (b)+(c)+(d)+(e). But term (a) is also active in the iteration process, where the model starts from arbitrary initial conditions (arbitrary length profiles of Q and y) and gradually (with steady boundary conditions) iterates towards the overall steady state situation.

The commercial computer models often give the user the option of choosing between the various approximations (as seen in table 2). By testing with different approximations and comparing the results, the user can obtain a better understanding of what hydraulic type the actual problem has. This understanding could be useful in the choice of solution for the actual problem.

2. Classification of wave phenomena in free surface flows

In order to be able to classify the unsteady physical flow type, this chapter will present tree basic types of one-dimensional wave phenomena. These are

1. Frictionless dynamic wave
2. Kinematic wave
3. Diffusive wave

These examples can be interpreted as theoretical extremities, which show how changes in water level and velocity are transported under various simplified conditions.

The common assumptions for the three cases is firstly that the continuity equqation is valid, which means that the water is incomrpressible. Secondly and in respect to the momentum equation, we assume that only longitudinal accelerations exist which is consistent to the assumption that the curvature of the streamlines is neglible which again corresponds to the assumption that hydrostatic pressure distribution perpendicular to the flow direction is present in all points.

Frictionless dynamic waves

Frictionless dynamic waves occur because of fast changes of the flow. Generally, this type of waves is only rarely seen in the unaffected nature. Most often, they are the result of man-made activities and are as examples seen after at a sudden start or stop of a pump station, a sudden change of flow in a hydropower station or a sudden opening of a gate. Frictionless dynamic waves are the fastest waves which can occur in watercourses (with subcritcal flow).

From basic hydraulics we know that a small and long shallow water wave in a horizontal channel has a celerity of either $c = +\sqrt{g\,D}$ or $c = -\sqrt{g\,D}$. ("Small" and "long"should be understood in relation to the water depth). From mathematics, we know that such a wave can be described by the so-called *wave-equation.*

$$\frac{\partial^2 \eta}{\partial x^2} = c\,\frac{\partial \eta}{\partial t}$$

where $\eta = \eta\,(x,t)$ is the level of the water surface relative to the mean water level.

The theory tells us that any continuous function of η is a solution to the wave-equation as long as the function moves translatoric with either $+$ c or - c . This means that a frictionless dynamic wave can have any form as long as it is small and long (compared to the water depth). Furthermore, the waves are linear and the principle of superposition is valid.

In practice, the assumption of a negible friction is only reasonable in a short time after the wave is formed. Later when the wave has moved further away, the influence of friction will gradually rise and after a certain time the designation *frictionless dynamic wave* does not cover the situation anymore.

Examples of frictionless dynamic waves

To calculate the wave height we can use the continuity equation plus the knowledge of the wave celerity $c = +\sqrt{g\,D}$. We are looking on a long horizontal channel (Figure 2-1) with a water depth of D and a width of B. Initially no flow takes place. The channel is closed as shown in the figure. Suddenly we start to pump in a constant flow of ΔQ at the closed end of the channel.

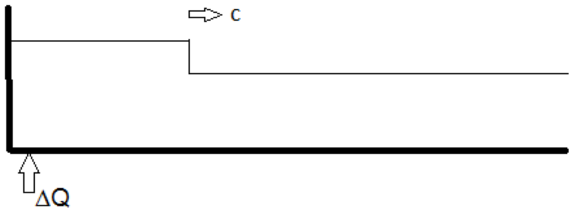

Figure 2-1 Frictionless dynamic wave

The continuity equation over the time increment Δt gives this equation:

$$c\,H\,B\,\Delta t = \Delta Q\,\Delta t$$

If the wave celerity c $= \sqrt{g\,D}$ is introduced we get this equation:

$$H = \frac{\Delta Q}{B\sqrt{g\,D}} \quad (1)$$

In the next case, we look at a similar channel where the water flows to the right with a constant speed V where the flow is subcritical (the Froude number < 1). Here a wave is formed which runs both to the left and the right with celerities of respectively c – V and c + V.

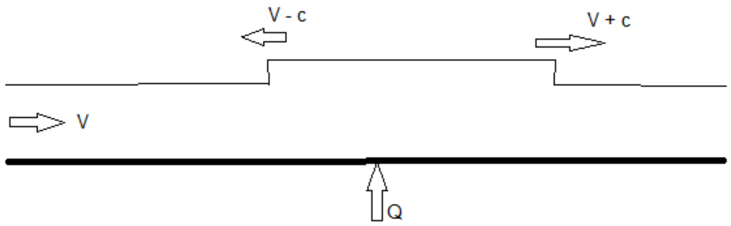

Figure 2-2 Snapshot of frictionless dynamic wave in channel with subcritical flow

Here, we can again use the continuity over a time increment Δt to find the wave height H:

$$(c - V)\,H\,B\,\Delta t + (V + c)\,H\,B\,\Delta t = \Delta Q\,\Delta t$$

which gives

$$H = \frac{\Delta Q}{2\,B\sqrt{g\,D}} \quad (2)$$

In the Danish landscape we often find lower agricultural areas along the watercourses from which drainage water is pumped up to the watercourses. Most often, the pump control system is a simple on/off control. Immediately after the pump start, an increase of the water level in the watercourse corresponding to formula (2) will occur, and the wave will run upstream and downstream as described above. After some time, the friction will take over and the water level will change as described in the following sections.

Hydro power stations, which often apply a high rate of the discharge in the watercourse, can generate significant changes in the flow in the connected watercourse because the control of turbines is connected to the electric power network. Accordingly, special arrangments for flow control are taken for safety reasons in hydropower systems to avoid flooding.

Kinematic waves

Most natural changes of water level and flow in watercourses take place relative slowly and can be described by the theory of *kinematic waves or diffusive waves*. The kinematic waves, which can be understood as a special case of diffusive waves, are described in this section. The designation ‘kinematic wave’ refers to a long wave which travels unchanged (translational) along the river with a constant celerity.

The changes in the flow and water level in the watercourses will normally not be experienced as a wave phenomenon. The understanding of a propagating wave behaviour appears first when time series of observed water levels from a number of locations along the water course later on are analyzed. An example of kinematic waves is seen in the Norwegian rivers caused by the melting snow when the frost breaks in the spring. These so-called *flom-waves* pass slowly over a couple of days through the valleys often with flooding as the consequence. The same thing is well known from the larger rivers in central Europe (a famous example is “Das Oderhochwasser 1997” in the river Oder). Because of the restricted length of the Danish streams, real (natural) flom-waves will only be seen partly.

These slow wave propagations are controlled by the hydraulic friction and stand physically as the opposition to the frictionless dynamic waves where inertia and acceleration are dominating as mentioned in the former section. As already mentioned is a *kinematic wave* is a very long wave of water level and flow which propagates unchanged along the watercourse with constant celerity.

The celerity c for a kinematic wave can be determined by watching a one-dimensional flow of Figure 2-3. The figure is out of scale. In the real world, a kinematic wave is much flatter.

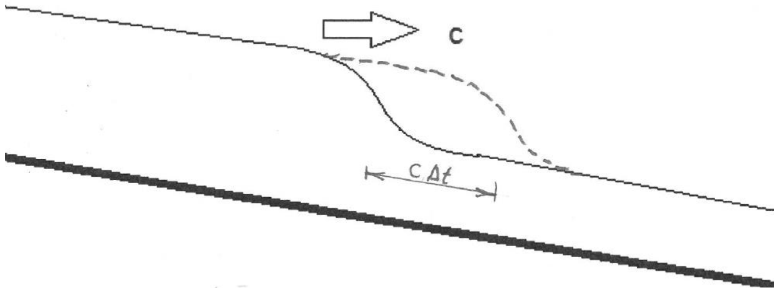

Figure 2-3 Propagation of kinematic wave

From the continuity equation, the celerity c is found:

$$\Delta Q\,\Delta t = c\,\Delta t\,\Delta A$$

where ΔA is the change of the cross-section area. This gives the equation:

$$c = \frac{\Delta Q}{\Delta A} \qquad (1)$$

and in differential form

$$c = \frac{dQ}{dA} \qquad (2)$$

If it is assumed that the cross-section is wide (with constant width B) and having constant depth y, and that furthermore the Manning formula $V = M\; y^{2/3}\, I^{1/2}$ can be applied, we get

$$c = \frac{d(V\,y\,B)}{d(y\,B)} = \frac{d(M\,y^{\frac{5}{3}}\,I^{\frac{1}{2}})}{dy} = \frac{5}{3}\,V$$

This is the wave celerity for small waves (small wave heights compared to water depth) in the unaffected flow. As seen the wave celerity is proportional to the flow velocity and, this way, dependent of water depth.

For other cross-sections, the wave celerity will depend on the type of cross-section as seen in Table 1.

Table 1 Kinematic wave celerity for various cross-sections

Cross-section	Kinematic wave celerity c
Rectangular	1,67 V
Triangular	1,33 V
Parabolic	1,44 V
Trapez	1,33 $V < c < 1{,}67\ V$

For real measured cross-section calculation becomes more complicated because both A and R vary with water depth. In this case, c can be found with a simple numerical differentiation from formula (1) by taking two water levels respectively y and $y + \Delta y$:

$$c = \frac{\Delta Q}{\Delta A} = \frac{Q(y + \Delta y) - Q(y)}{A(y + \Delta y) - A(y)}$$

As a rule of thumb, we can assume that the wave celerity for slow variations in water level and flow is a factor 1,5 times the average flow velocity. This means that the “flow-wave” from a sudden short discharge of a slug of polluted water travels faster than the “pollution-wave” does.

A kinematic wave propagate theoretically unchanged along the watercourse with the celerity mentioned above. The kinematic wave only travels forward in the flow direction.

Diffusive wave

Because of the formal similarities between the description of diffusive waves (waves of water level and flow) and the corresponding description of transport and dispersion of waves of soluble matters, we will first give a short presentation of the last-mentioned issue (which will not further be mentioned in these notes).

Transport and dispersion of a ”concentration-wave” of soluble matter

For the transport and dispersion of concentration of soluble matters in a one-dimentional, steady, uniform flow where the concentration does not give changes in volume, we can apply the so-called *transport/dispersion equation* (equation (4) below). This equation is developped from Fick´s first law (linear diffusive transport) and the continuity equation for the solubles. Be aware that we here (and only in this sub-section) use the letter c for concentration of soluble matters:

$$\frac{\partial c}{\partial t} + V\,\frac{\partial c}{\partial x} = D\,\frac{\partial^2 c}{\partial x^2} \quad (4)$$

where V is flow velocity and D is the dispersion coefficient.

The solution to equation (4) after a time $t = t_1$ if we assume that the initial condition is that the total mass M of the soluble matter is gathered in point x= 0 at time t=0 (known as the *Dirac delta function*) is given by formula (5).

$$c = \frac{M}{\sqrt{4\,\pi\,D\,t}}\,e^{\left(\frac{-(x - V\,t)^2}{4\,D\,t}\right)} \quad (5)$$

Figure 2-4 shows the solution to equation (4) for $t = 0, t_1$ and t_2

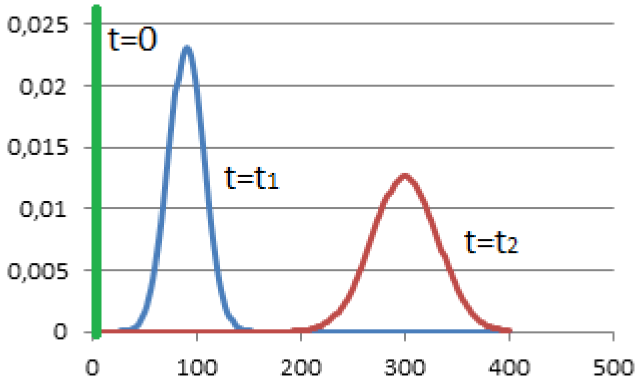

Figure 2-4 Graph of solutions to the transport/diffusion equation

Transport and diffusion of flow-wave

The diffusive wave is described by the simplified Saint-Venant equations where the two acceleration terms are excluded. This way the momentum equation espresses a balance between the longitudinal pressure gradient (from the surface slope) and the hydraulic friction. The equations are as follows:

continuity

$$\frac{\partial Q}{\partial t} + B\frac{\partial y}{\partial x} = q_L$$

momentum

$$\frac{\partial h}{\partial x} - \left(I_0 - I_f\right) = 0$$

where $h = z + y$ is the surface level (z is bottom level and y is water depth), I_0 slope of bottom, and I_f is the friction slope (for example given by the the Manning formula $Q = A\,M\,R^{2/3}\,I_f^{1/2}$).

Under the assumption that the wave only gives small changes of flow and water level in a prismatc channel the two equations can be coubled together (Schaarup-Jensen, 1987). The formal elaboration will not be given here, but the result is that both the change of flow and the change of surface level are given by the transport/dispersion equation:

$$\frac{\partial y}{\partial t} + c\,\frac{\partial y}{\partial x} = D_F\frac{\partial^2 y}{\partial x^2} \quad (2) \quad \text{water level}$$

$$\frac{\partial Q}{\partial t} + c\,\frac{\partial Q}{\partial x} = D_F\frac{\partial^2 Q}{\partial x^2} \quad (3) \quad \text{flow}$$

where c is the kinematic wave celerity $c = dQ/dA$ as mentioned in the former section The dispersion coefficient for the flow changes D_{flow} is found to be

$$D_{flow} = \frac{Q_0}{2\ B\ I_f}$$

where Q_0 is the unaffected steady flow and I_f is the friction slope (e.g. found from the Manning formula).

As seen, equations (2) and (3) are linear differential equations, which are only valid for small changes. Larger diffusive waves are not linear and should be calculated by numerical methods. A following chapter will give an example of such a numerical model.

Almost all changes of flow and water level caused by hydrological changes (from precipitation, snow melting etc.) in Danish watercourses are quite well described by the theory for diffusive waves. Even the intrusion of tidal waves from the sea and waves from overflows from urban catchments, are well described by this theory.

3. Numerical modelling of non-steady free surface flows

There is an almost inifinite number of numerical methods for the modelliling of non-steady free surface flows. Only the most simple and direct method building on *finite differences* and the *control volume method* in staggered grid is presented here as an introduction to the complicated subject. This method is a so-called explicit method because the equations are arranged in a way that the new unknown values can be calculated direct (explicitly) from old known values. The advantage of explicit models is their simple structure which makes them fast and easy to set up. The disadvantage is that these models require small steps in time to obtain numerical stability.

Commercial models are almost always implicit models which solve the complete Saint Venant equations and can give stable and accurate results in most cases. The quality of the results from the commercial models depends primarily on the quality of the configuration of the model (especially the choice of time step and space step) and the input data. Accordingly, it is recommended to carry out an system analysis in order to find the proper configuration for the model.

Controle volume method in staggered grid

In principel we have two unknows in the modelling: The depth averaged flow velocity V and the level h of the surface both as functions of time and space. (Alternativly the unknows could be the flow Q and the water depth y). Corresponding we have two governing equations: The continuity and the momentum equations. When we discretize the equations and set up the difference equations in a computational grid (including also the initial and the boundary conditions) we should in principle ends up with the same number of unknowns as the total number of equations.

Finite differences

The numerical solution calculates the 2-dimensional functions $V(x,t) = V_j^n$ and $h(x,t) = h_j^n$ in discrete points (j Δx, n Δt) in time and space, where j is the distance-index and n is the time-index, both are integers with values of 1, 2, 3 etc.

The governing equations are discretized by the substitution of the differential derivatives by finite difference equations.(For practical reason are only the derivaties related to V are shown here). Here simple forward differences are aplied (comercial models employ more complex and accurate equations):

$$\frac{\partial V}{\partial x} \cong \frac{\Delta V}{\Delta x} = \frac{V_{j+1}^n - V_j^n}{\Delta x} \quad \text{and} \quad \frac{dV}{dt} \cong \frac{\Delta V}{\Delta t} = \frac{V_j^{n+1} - V_j^n}{\Delta t}$$

Computational grid

The computation takes place in an outer time loop where the time progress with a time step Δt and inside the time loop run two parallel distance loops:

> Loop 1. New unknown values of velocity V_j^n (or Q) are determined by the momentum equation from known values of flow and head.
>
> Loop 2. New values of head h_j^n (or y) are calculated by the continuity equation from known values of flow and head (also including values just calculated in loop 1).

(Remark that the *sequence* of the two steps is arbitrary).

By splitting up the computations this way, it is obvious that the values of V and h (or Q and y) do not exist exactly at the same time step. They are staggered with $\Delta t/2$ from each other. Similar are the values staggered in distance with Δx/2 along the x-axis. This make the numbering of the elements a little bit confusing because the time indeks n refers both to flow at time n Δt and head at time (n+½) Δt as seen on the t/x-diagram (time/distance diagram) below (Figure 3-1).

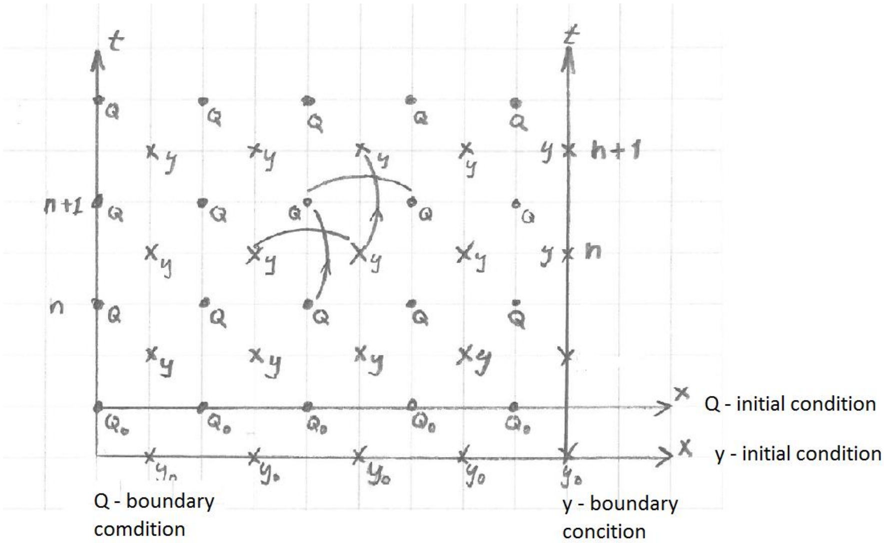

Figure 3-1 x/t-diagram of computational grid

With this type of model we should start with either an V-point (or Q-point) and end with a h-point (or y-point) or the other way around. It is not hydraulically correct (or possible) to specify both values at both boundaries.

Initial and boundary conditions

It is obvious that the model should have a complete set of *V* and *h* (or Q and y) values as initial conditions. Quite often we want the modelling to start from a steady situation and instead of using time for calculating this in advance, it is easier simply to let the model find the steady situation itself by running for some time with steady boundary conditions starting from a simple arbitrary guess of the initial values.

Example of a simple numerical model based on the diffusive wave assumption

In this example we want to calculate the non-steady hydraulic effect in a river when a short time limited lateral inflow suddenly occurs. The lateral inflow could, for example, be a storm overflow for an urban catchment.

The watercourse has a constant rectangular cross-section with a width of *B* and a steady bed slope of *I*.

Figure 3-2 Shows the length profile of the watercourse and the controle volume for each grid cell.

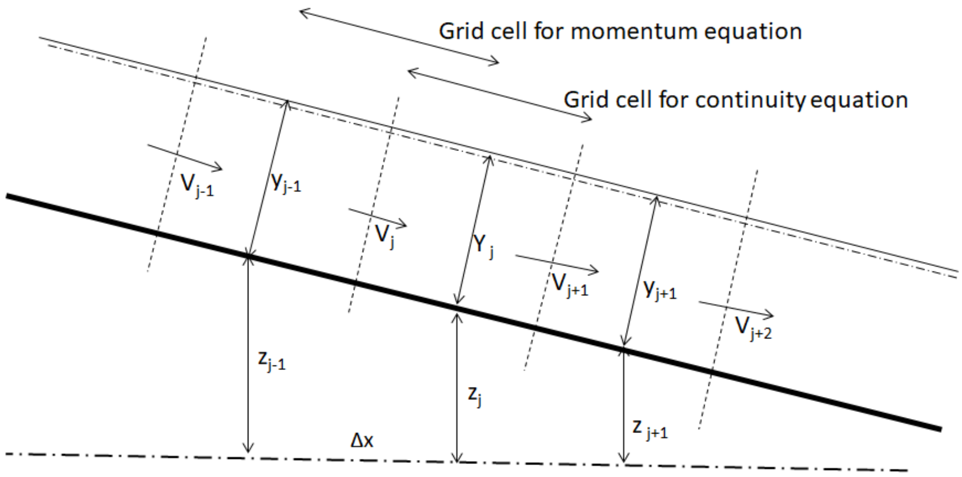

Figure 3-2 Length profile and computational grid. The grid cells for the continuity equations and the grid cells for the momentum equations are staggered with Δx/2. (Slope is strongly exaggerated in figure)

Continuity equation

Storage in grid cell = Inflow – outflow + lateral inflow [per unit time]

$$\Delta y_j = V_j\, A_j - V_{j+1} + S_j$$

Momentum equation

For the case of the diffusive wave approximation, the momentum equation reduces to the Manning equation (or a similar equation which contains the hydraulic friction):

$$V = M\, R^{2/3}\, I^{1/2}$$

$$V_j^n = M\, \left(R_j^n\right)^{2/3} \left[\frac{\left(y_j^n + z_j\right) - \left(y_{j-1}^n + z_{j-1}\right)}{\Delta x}\right]^{1/2}$$

$$Q_j^n = A_j^n\, V_j^n$$

Code of diffusive wave model for water course receiving an unsteady lateral inflow

Appendix A provides the computer code for the flow in a water course (simplified to a straight channel with rectangular cross-section) receiving an unsteady lateral inflow (without momentum). The code is written in *Pascal* and should easily be transformed to any other language. Notice: In standard Pascal the exponential functions have to be written as $a^b = e^{b \ln a}$.

Results from modelling

Data for modelling is written in the beginning of the program. The steady state flow is 0.3 m^3/s . The lateral inflow of 0,2 m^3/s comes into station 1000 m and last for 600 s. Figure 3-3 and Figure 3-4 shows results for water depth and flow in station 950 m (50 m upstream the lateral inlet) and in station 1250 (250 m downstream lateral inlet).

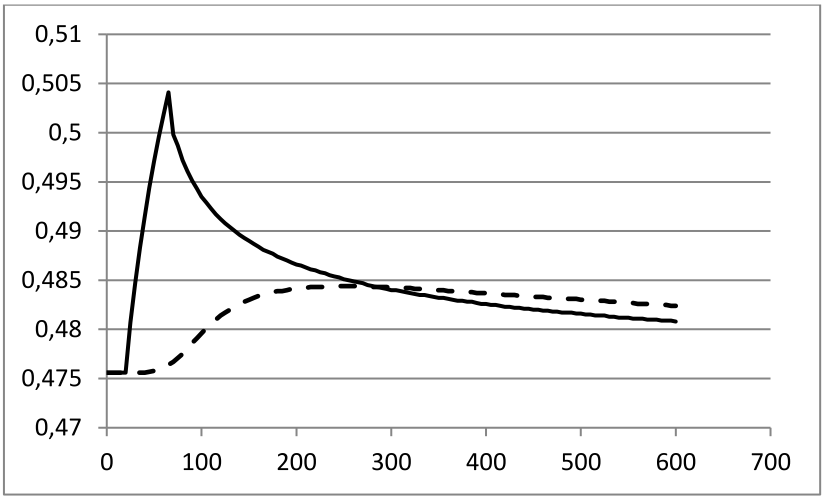

Figure 3-3 Time series of water depth in station 950 m (full line) and in station 1250 m (dotted line)

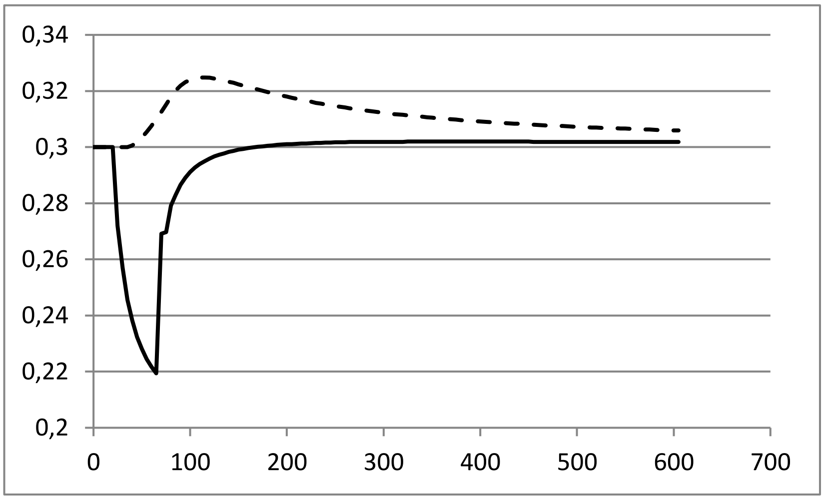

Figure 3-4 Time series of flow in station 950 m (full line) and in station 1250 m (dotted line)

In order to make this example as simple as possible it has be chosen to allow the lateral inflow rise momentarily up to relatively high value, which then is kept constant a certain time. However, it should be mentioned that this could cause local accelerations, which not are covered very well by the diffusive wave approximation.

4. Special examples of unsteady flows

Stability of steady uniform free surface flows – roll waves

Under certain circumstances will steady uniform flows in open channels will become unstable and thus non-stationarty. From practical experiences, it is known that the flow becomes unstable when the river is very steep or when the velocity is very high. Unstable flow is connected with super-critical flow (Froude number larger than 1). The unstable flow is expressed with the formation of breaking waves on the water surface, the so-called roll waves (Figure 4-1).

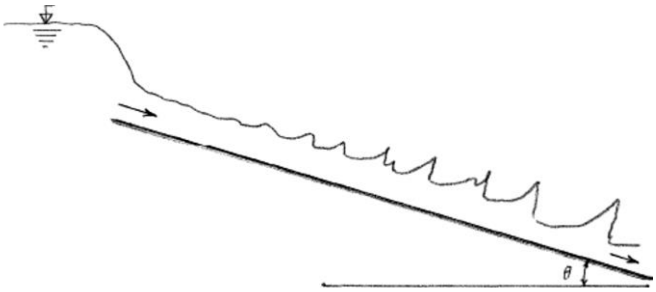

Figure 4-1Formation of roll waves

A full description on the criterion for the formation of roll waves is complicated and will not be given here. It schould only be mentioned that this is about the relation between the dynamic wave celerity and kinematic wave celerity. The transition from stable on unstable flow in rectangular channel takes place when the *Froude number* passes a value of 2.0.

It is a general experience from natural lowland watercourses, that the flow runs slowly with subcritical speed, meaning that the dynamic wave celarity is larger than the kinematic wave celerity. Accordingly roll-waves can not be seen in lowland rivers and streams. It is a self-regulating mechanism where any occurance of high flow velocities will cause a high erosion of soft sediments which then wil reduce the velocity.

In contrast to natural lowland water courses artificial (man-made) channels can be constructed with steep slopes where supercritical flow can occur and where the flow can be unstable. The risk of unstable flows in channels and in free surface pipeline flows is often enhanced because the hydraulic roughness here is low and low roughness leads to higher velocities. Unstable flow in pipelines (for example sewers) which are near from running full, can form very complex flow (and even dangerous) flow patterns. (Yen and Pansic, 1980).

A comprehensive scientific literature on flow stability can be found. A starting point could be Chow (1959), Ponce (1992) and Sjöberg (1982).

Dam-break waves

A sudden breakdown of a high dam in front of a water reservoir can have catastrophic consequences for the downstream areas of land. Accordingly, the establishment of such dams always includes profound investigations of the possible floods created of the dam-break wave flushing through the downstream landscape. Often both physical and numerical modeling are applied.

Tidal bores

The tide is formed in the oceans where the water depth is several kilometers (the average depth of the oceans is about 3.500 meters). Here, the tidal waves have a height (from trough to crest) of approximately 0.5 m. When the waves approach the coast, a transformation takes place.These are the following:

- Reflection
- Shoaling which means that the waves height and length because of the reduction of the water depth
- Diffraction which is the change of direction
- Increase of wave height because the cross-section area in the river mouth is narrowed

The changes can cause a steep wave which could break and entail a formation of a so-called tidal bore in the first part of the river. Hydraulically, the bore can be interpreted as moving hydraulic jump.

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5. References

This list of references only shows a few examples of literature on the comprehensive subject area.

Chow, V. T. (1959). Open.Channel Hydraulics., McGraw-Hill, New York.

Ponce, V: M. (1991). New perspective of the Vedernikov number. Water Resources Research, Vol. 27, pages 1777-1779, July 1991.

Schaarup-Jensen, K. (1987). Ikke-stationære vandløbsstrømninger og numerisk kinematisk bølgemodel. (Unsteady flows in watercourses and numerical kinematic wave model). In Danish. Lecture notes from Hydraulic Division, Departmet of Civil Engineering, Aalborg University.

Sjöberg, A. (1982). On the stability of gradually varied flows in sewers. Report No 3061. Deartment of Water Ressources Engineeering. Chalmers University of Technology, Gothenburg.

Yen, B. C., Pansic, N. (1980). Surcharge of sewer systems. WRC Research Report No. 149, University of Illinois.

Appendix A Computer code for diffusive wave model

```
program difusivewave;
{Unsteady river model with lateral inflow
diffusive wave approximation - Code in Pascal - Torben Larsen 2018 }

uses crt;

Const                {Data for  calculation}
 Imax=200;           {number of node points}
 dx=50.0;            {space step m}
 dt=5.0;             {time step s}
 Tmax=2400.0         {time of simulation}
 Q0=0.300;           {steady state input flow at upstream boundary m3/s}
 z0=40;              {level of river bottom at upstream boundary m}
 B=2.0;              {width of river m}
 Ibottom=0.0005;     {river slope m/m}
 M=30.0;             {Manning number m(1/3/s)}
 Iside=20;           {Node point number where lateral inlet takes place}
 Qside=0.3;          {lateral inflow m3/s}
 T1side=20.0;        {time when lateral infloe starts s}
 T2side=60.0;        {time when lateral inflow stops s}
 out1=19;            {node point number for output 1 }
 out2=25;            {node point number for output 2 }

Var {Declaration of variables}
i                                :integer;
t,slope,error,y0,Qs,ymax,ymin    :real;
y,z,h,Q,A,R                      :array[1..Imax] of real;
udfil                            :text;

Begin
Clrscr;
Assign(udfil, 'Qmod.res'); Rewrite(udfil);

{iteration for steady state water depth}
ymax:=1000; ymin:=0.0;
Repeat
   y0:=0.5*(ymax+ymin);
   error:=(y0*b)*M*exp((2/3)*ln((y0*b)/(b+2*y0)))*sqrt(Ibottom)-Q0;
   If error>0.0 Then ymax:=y0;
   If error<0.0 Then ymin:=y0;
Until (abs(error)<0.00000001);
```

```
y0:=0.5*(ymax+ymin);
```

(continue)

```
For i:=1 to imax do {Initial conditions}
  Begin
     z[i]:= z0-(i-0.5)*dx*Ibottom;
     y[i]:= y0;
     Q[i]:= Q0;
   End;

t:= 0.0;
Repeat {Begin of time loop}

Q[1]:=Q0; y[imax]:=y0; {boundary conditions}

For i:=1 to (Imax-1) do   {length profile loop - water depth}
  Begin
      Qs:=0.0; if (i=Iside) and (t>=t1side) and (t<=t2side) then Qs:=Qside;
      y[i]:=y[i]+(Q[i]-Q[i+1]+Qs)*dt/(b*dx);
  End;

For i:=2 to Imax do  {length profile loop - flow}
  Begin
     A[i]:=b*y[i];    R[i]:=b*y[i]/(b+2*y[i]);
     Slope:= ((y[i-1]+z[i-1])-(y[i]+z[i]))/dx;

     If Slope>=0.0 then Q[i]:= A[i]*M*exp((2/3)*ln(R[i]))*sqrt(Slope);
     If Slope< 0.0 then Q[i]:=-A[i]*M*exp((2/3)*ln(R[i]))*sqrt(-Slope);
  End;

  Qs:=0.0; if (t>=t1side) and (t<=t2side) then Qs:=Qside;
  Writeln(udfil,t:6:0,'  ',y[out1]:6:4,' ',y[out2]:6:4,'    ',t:6:0,' ',Q[out1]:6:4,'  ',Q[out2]:6:4);
  Flush(udfil);
t:=t+dt;
Until t>Tmax; {end of time loop}
End.   {end of program}
```

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